

NAG Toolbox for MATLAB

s15af

1 Purpose

s15af returns a value for Dawson's Integral, $F(x)$, via the function name.

2 Syntax

```
[result, ifail] = s15af(x)
```

3 Description

s15af evaluates an approximation for Dawson's Integral

$$F(x) = e^{-x^2} \int_0^x e^{t^2} dt.$$

The function is based on two Chebyshev expansions:

For $0 < |x| \leq 4$,

$$F(x) = x \sum_{r=0}^l a_r T_r(t), \quad \text{where} \quad t = 2\left(\frac{x}{4}\right)^2 - 1.$$

For $|x| > 4$,

$$F(x) = \frac{1}{x} \sum_{r=0}^l b_r T_r(t), \quad \text{where} \quad t = 2\left(\frac{4}{x}\right)^2 - 1.$$

For $|x|$ near zero, $F(x) \simeq x$, and for $|x|$ large, $F(x) \simeq \frac{1}{2x}$. These approximations are used for those values of x for which the result is correct to **machine precision**. For very large x on some machines, $F(x)$ may underflow and then the result is set exactly to zero.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result** – double scalar

The result of the function.

2: **ifail** – int32 scalar

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

There are no failure exits from this routine.

7 Accuracy

Let δ and ϵ be the relative errors in the argument and result respectively.

If δ is considerably greater than the *machine precision* (i.e., if δ is due to data errors etc.), then ϵ and δ are approximately related by:

$$\epsilon \simeq \left| \frac{x(1 - 2xF(x))}{F(x)} \right| \delta.$$

The following graph shows the behaviour of the error amplification factor $\left| \frac{x(1 - 2xF(x))}{F(x)} \right|$:

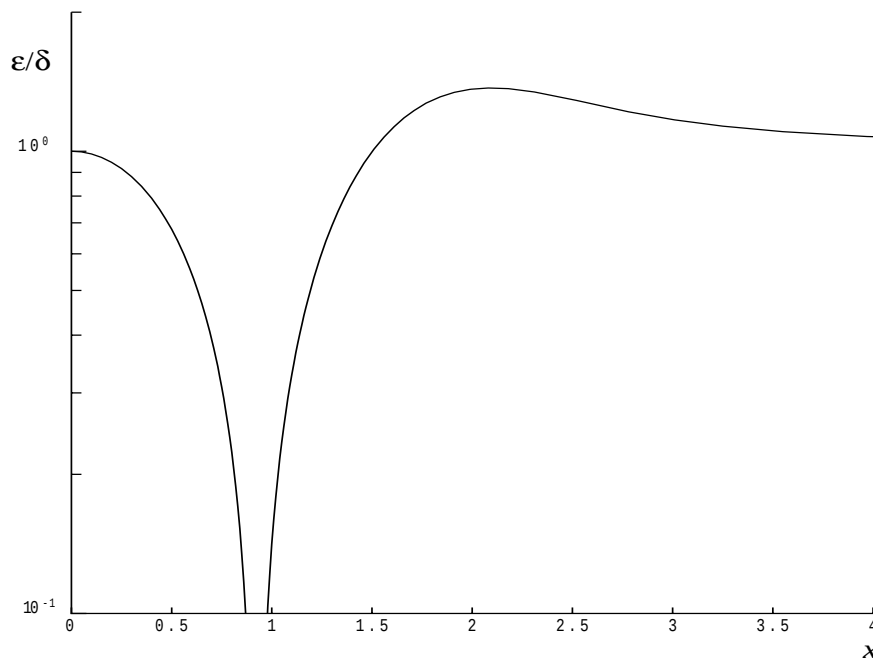


Figure 1

However if δ is of the same order as *machine precision*, then rounding errors could make ϵ somewhat larger than the above relation indicates. In fact ϵ will be largely independent of x or δ , but will be of the order of a few times the *machine precision*.

8 Further Comments

None.

9 Example

```
x = -2;  
[result, ifail] = s15af(x)  
  
result =  
    -0.3013  
ifail =  
         0
```
